DYNAMIC STRESS AMPLITUDE IN A FAST PULSED REACTOR

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Inzhenerno-Fizicheskii Zhurnal, Vol. 14, No. 1, pp. 134–137, 1968

UDC 536.48

The dynamic stresses in the core components of a fast pulsed reactor are investigated. The results of a numerical calculation are presented.

In fast pulsed reactors operating on the thermal expansion principle the energy release is limited by the dynamic stresses that develop during the pulse in the core material [1, 2]. In most cases in selecting the reactor rating it is sufficient to know only the maximum value (amplitude) of the stresses, which is reached in the free vibration stage [1, 3].

In the core components of a fast pulsed reactor the stress amplitude depends only on the total temperature rise during the pulse and on the product of the natural frequencies of the component and the pulse width [4]. The stresses are almost insensitive to other finer variations in pulse shape.

This paper presents the results of numerical calculations intended to verify these conclusions.

The calculations employ the complete set of pulses obtained in the simple case when the reactor processes are determined by a single oscillatory system with one degree of freedom. The dependence of stress amplitude on pulse shape is investigated with reference to the example of spherical shells with different natural frequencies. The ratio of the effective stresses in the shell to the stresses that would occur in the total absence of thermal expansion of the shell (the quantity

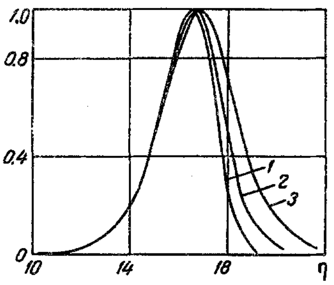

Fig. 1. Shape of power pulse at various values of the parameter $\omega_1\tau_0$: 1) $\omega_1\tau_0 = 0.18$; 2) 1.44; 3) 11.5.

F) is computed. This ratio best characterizes the dynamics of the thermal expansion of the shell.

The system of equations has the form

$$\frac{dn}{dt} = \left(\frac{1}{\tau_0} - \gamma_1 u_1\right) n,$$

$$\frac{d^2u_i}{dt^2} = \omega_i^2(\alpha_i q - u_i), \quad q(t) = \int_0^t n(t)\,dt,$$

$$F_i(t) = \left(\frac{u_i}{\alpha_i q} - 1\right), \quad i = 1,\ 2,\ 3. \tag{1}$$

The equations were solved numerically on a computer for the following values of the parameters: $\gamma_1 = 2\cdot 10^9$

Stress Amplitude F° Obtained from the Analytic Formula and the Numerical Calculations

ωT	Stress amplitude F°			
	Calculated from (4)	Numerical calculations		
		$\omega=\omega_1$	$\omega=\omega_2$	$\omega=\omega_3$
0.156	0.997	—	0.997	—
0.314	0.987	—	0.987	—
0.468	0.973	0.972	—	—
0.645	0.947	—	0.949	—
0.940	0.892	0.893	—	—
1.380	0.786	—	0.784	—
1.404	0.780	—	—	0.787
1.935	0.636	0.65	—	—
2.82	0.410	—	—	0.44
3.21	0.329	—	0.32	—
4.14	0.184	0.19	—	—
5.81	0.059	—	—	0.08
6.72	0.030	—	0.03	—
9.62	0.003	0.004	—	—

1/sec·m; $\omega_1 = 0.45\cdot 10^5$, $\omega_2 = 0.15\cdot 10^5$, $\omega_3 = 1.35\cdot 10^5$ 1/sec; $\alpha_1 = 0.75\cdot 10^{-4}$, $\alpha_2 = 2.25\cdot 10^{-4}$, $\alpha_3 = 0.25\cdot 10^{-4}$ m/MJ; $\tau_0 = 4\cdot 10^{-6}$, $8\cdot 10^{-6}$, $16\cdot 10^{-6}$, $32\cdot 10^{-6}$, $64\cdot 10^{-6}$, $128\cdot 10^{-6}$, $256\cdot 10^{-6}$ sec.

In the case represented by Eqs. (1) the pulse shape depends only on the parameter $\omega_1\tau_0$. As it decreases, the half-power pulse width, expressed in τ_0 units, decreases and the pulse becomes increasingly asymmetric.

At large values of $\omega_1\tau_0$ ($\omega_1\tau_0 > 3$) inertia effects become unimportant and can be neglected. In this case the second of Eqs. (1) becomes

$$u_1 = \alpha_1 q,$$

and system (1) is solved analytically. If the time is measured in τ_0 units and reckoned from the maximum of the power n, the solution is

$$q(\eta) = \frac{q(\infty)}{1 + e^{-\eta}}, \quad T = 3.5255\tau_0. \tag{2}$$

Here the amplitude of F(t) in the free vibration stage is determined from the analytic expression

$$F^0 = \frac{\tau_0\omega\pi}{\operatorname{sh}\tau_0\omega\pi}. \tag{3}$$

If our initial assumption concerning the dependence of stress amplitude on pulse shape is correct, on the basis of (3) for a pulse of arbitrary shape we obtain

$$F^0 = \frac{T\omega\pi}{3.5255}\left(\operatorname{sh}\frac{T\omega\pi}{3.5255}\right)^{-1}. \tag{4}$$

The results of the calculations are presented in Figs. 1–3 and in the table. In Fig. 1 the power pulse

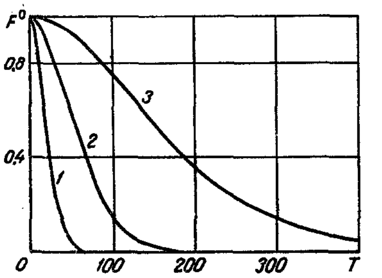

Fig. 2. Amplitude of F(t) as a function of pulse width for spherical shells with different vibration frequencies (T in μsec): 1) $\omega = 1.35 \cdot 10^5$ 1/sec; 2) $0.45 \cdot 10^5$; 3) $0.15 \cdot 10^5$.

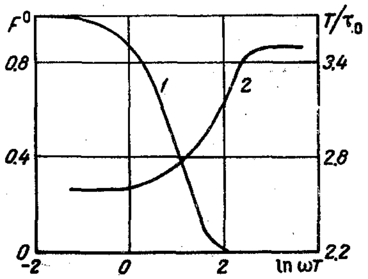

Fig. 3. Amplitude of F(t) as a function of the product $\omega_1 T$ (curve 1) and pulse width in τ_0 units as a function of the product $\omega_1 T$ (curve 2).

shape is shown for the three most characteristic values of $\omega_1 \tau_0$. In Fig. 2 the amplitude of F(t) is given as a function of the pulse width for three spherical shells with different vibration frequencies. It is clear from Fig. 2 that as the pulse width increases the amplitude of F(t) falls the more steeply from unity to zero, the greater the vibration frequency of the shell. In Fig. 3 the amplitude of F(t) is presented as a function of $\omega_1 T$, and the pulse width in τ_0 units, which characterizes the change in pulse shape, as a function of $\omega_1 T$. To the scale selected the values of F° for each shell lie close to the same curve.

In the table the amplitudes calculated from (4) are compared with the numerical solution of Eqs. (1).

Clearly, the stress amplitudes for each of the shells obtained from numerical calculations are very close to the values determined from (4). This fact and the data of Fig. 3 show that in estimating the stresses in a fast pulsed reactor it is indeed possible to assume that the stress amplitude depends only on the total heating of the components during the pulse and on the product of the natural frequencies of the component and the pulse width.

Finally we present formulas for estimating the stress amplitude in components of considerable thickness.

If for these components we employ the solution of the thermoelastic problem in the form of a Fourier series, the stresses in the one-dimensional case have the form [4]

$$\sigma(x, t) = A_0(x) q(t) + \sum_{i=1}^{\infty} A_i(x) \int_0^t q(z) \sin \omega_i (t - z)\, dz, \tag{5}$$

where $A_0(x)$, $A_i(x)$ are functions depending on the space coordinates.

In analogy with the derivation of Eq. (4) we obtain the following estimate for the stress amplitude at point x:

$$\frac{\sigma^0(x)}{q(\infty)} \simeq \left| A_0(x) + \sum_{i=1}^{\infty} \frac{A_i(x)}{\omega_i} \right| + \sum_{i=1}^{\infty} \left| \frac{A_i(x)}{\omega_i} \, \frac{T \omega_i \pi}{3.5255} \, \frac{1}{\operatorname{sh} \frac{T \omega_i \pi}{3.5255}} \right|. \tag{6}$$

NOTATION

n(t) is the reactor power; n_M is the maximum power; q(t) is the energy released in the reactor by time t; $q(\infty)$ is the value of q at the end of the pulse; t is the time; τ_0 is the initial reactor going-up period; T is the half-power pulse width; $\eta = t/\tau_0$; ω_i, u_i are the cyclic vibration frequency and the radial displacement of the i-th shell, respectively; α_i is a parameter depending on dimensions, heat capacity, and coefficient of thermal expansion of the i-th shell; γ_1 is the reactivity coefficient; x is a space coordinate; and $\sigma(x, t)$, $\sigma^0(x)$ are the stress in the component at point x and time t and the maximum stress, respectively.

REFERENCES

1. V. F. Kolesov, Atomnaya energiya, **14**, 273, 1963.

2. T. Wimett, Paper SM 62/53 (USA) at IAEA Symposium, Karlsruhe, May 1965.

3. D. Burgreen, Nuclear Sci. and Engng., **12**, 203, 1962.

4. V. F. Kolesov, Atomnaya energiya, **20**, 265, 1966.

13 March 1967